3 | INTERFERENCE

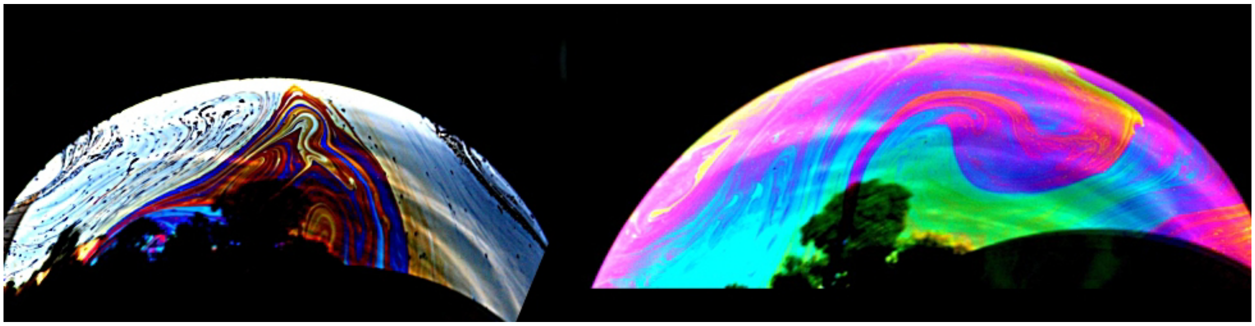

Figure 3.1 Soap bubbles are blown from clear fluid into very thin films. The colors we see are not due to any pigmentation but are the result of light interference, which enhances specific wavelengths for a given thickness of the film.

Chapter Outline

Introduction

The most certain indication of a wave is interference. This wave characteristic is most prominent when the wave interacts with an object that is not large compared with the wavelength. Interference is observed for water waves, sound waves, light waves, and, in fact, all types of waves.

If you have ever looked at the reds, blues, and greens in a sunlit soap bubble and wondered how straw-colored soapy water could produce them, you have hit upon one of the many phenomena that can only be explained by the wave character of light (see **Figure 3.1**). The same is true for the colors seen in an oil slick or in the light reflected from a DVD disc. These and other interesting phenomena cannot be explained fully by geometric optics. In these cases, light interacts with objects and exhibits wave characteristics. The branch of optics that considers the behavior of light when it exhibits wave characteristics is called wave optics (sometimes called physical optics). It is the topic of this chapter.

3.1 | Young's Double-Slit Interference

Learning Objectives

By the end of this section, you will be able to:

- Explain the phenomenon of interference
- Define constructive and destructive interference for a double slit

The Dutch physicist Christiaan Huygens (1629–1695) thought that light was a wave, but Isaac Newton did not. Newton thought that there were other explanations for color, and for the interference and diffraction effects that were observable at the time. Owing to Newton's tremendous reputation, his view generally prevailed; the fact that Huygens's principle worked was not considered direct evidence proving that light is a wave. The acceptance of the wave character of light came many years later in 1801, when the English physicist and physician Thomas Young (1773–1829) demonstrated optical interference with his now-classic double-slit experiment.

If there were not one but two sources of waves, the waves could be made to interfere, as in the case of waves on water (**Figure 3.2**). If light is an electromagnetic wave, it must therefore exhibit interference effects under appropriate circumstances. In Young's experiment, sunlight was passed through a pinhole on a board. The emerging beam fell on two pinholes on a second board. The light emanating from the two pinholes then fell on a screen where a pattern of bright and dark spots was observed. This pattern, called fringes, can only be explained through interference, a wave phenomenon.

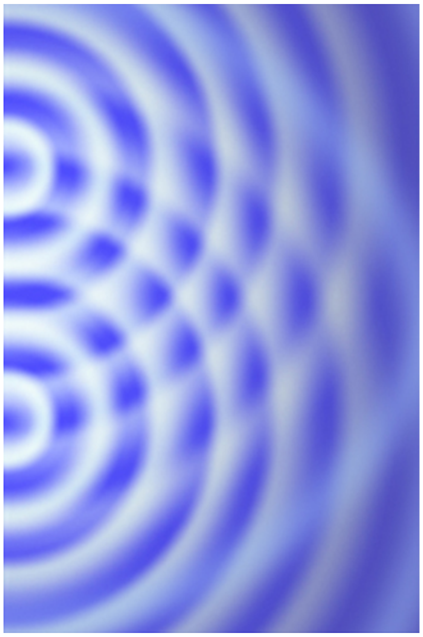

Figure 3.2 Photograph of an interference pattern produced by circular water waves in a ripple tank. Two thin plungers are vibrated up and down in phase at the surface of the water. Circular water waves are produced by and emanate from each plunger. The points where the water is calm (corresponding to destructive interference) are clearly visible.

We can analyze double-slit interference with the help of **Figure 3.3**, which depicts an apparatus analogous to Young's. Light from a monochromatic source falls on a slit S_0. The light emanating from S_0 is incident on two other slits S_1 and S_2 that are equidistant from S_0. A pattern of *interference fringes* on the screen is then produced by the light emanating from S_1 and S_2. All slits are assumed to be so narrow that they can be considered secondary point sources for Huygens' wavelets (**The Nature of Light**). Slits S_1 and S_2 are a distance *d* apart ($d \leq 1 \text{ mm}$), and the distance between the screen and the slits is $D(\approx 1 \text{ m})$, which is much greater than *d*.

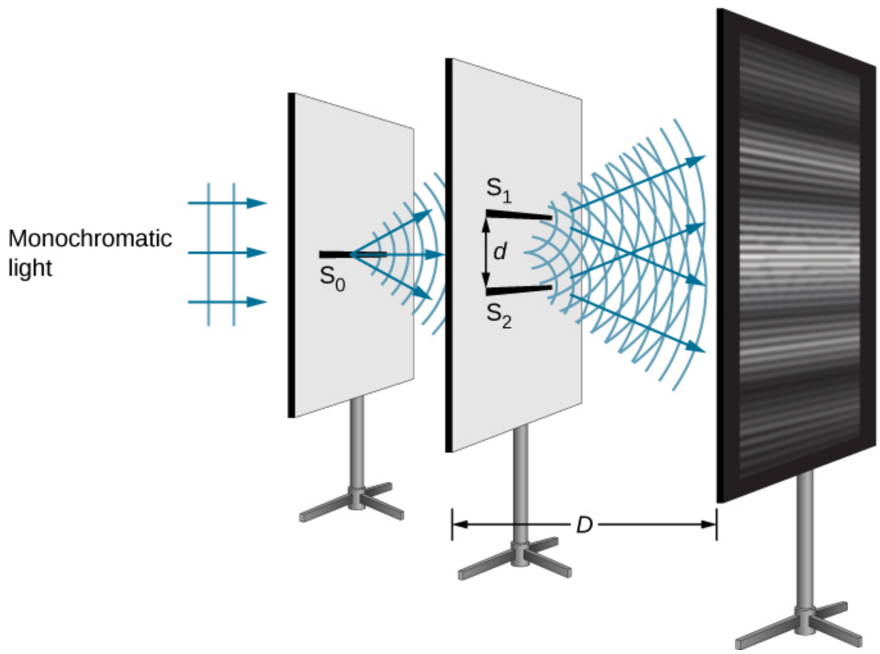

Figure 3.3 The double-slit interference experiment using monochromatic light and narrow slits. Fringes produced by interfering Huygens wavelets from slits S_1 and S_2 are observed on the screen.

Since S_0 is assumed to be a point source of monochromatic light, the secondary Huygens wavelets leaving S_1 and S_2 always maintain a constant phase difference (zero in this case because S_1 and S_2 are equidistant from S_0) and have the same frequency. The sources S_1 and S_2 are then said to be coherent. By **coherent waves**, we mean the waves are in phase or have a definite phase relationship. The term **incoherent** means the waves have random phase relationships, which would be the case if S_1 and S_2 were illuminated by two independent light sources, rather than a single source S_0. Two independent light sources (which may be two separate areas within the same lamp or the Sun) would generally not emit their light in unison, that is, not coherently. Also, because S_1 and S_2 are the same distance from S_0, the amplitudes of the two Huygens wavelets are equal.

Young used sunlight, where each wavelength forms its own pattern, making the effect more difficult to see. In the following discussion, we illustrate the double-slit experiment with **monochromatic** light (single λ) to clarify the effect. **Figure 3.4** shows the pure constructive and destructive interference of two waves having the same wavelength and amplitude.

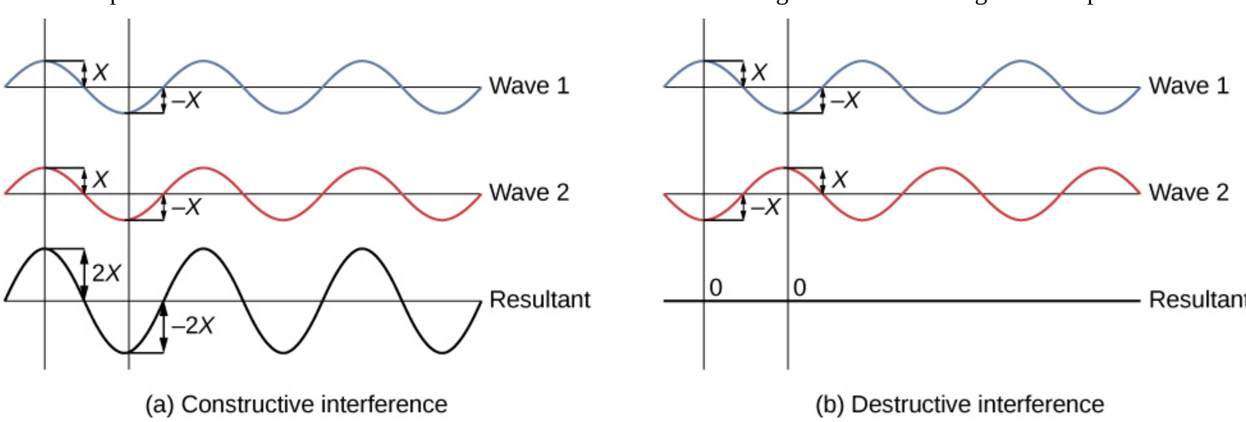

Figure 3.4 The amplitudes of waves add. (a) Pure constructive interference is obtained when identical waves are in phase. (b) Pure destructive interference occurs when identical waves are exactly out of phase, or shifted by half a wavelength.

When light passes through narrow slits, the slits act as sources of coherent waves and light spreads out as semicircular waves, as shown in **Figure 3.5**(a). Pure *constructive interference* occurs where the waves are crest to crest or trough to trough. Pure *destructive interference* occurs where they are crest to trough. The light must fall on a screen and be scattered into our eyes for us to see the pattern. An analogous pattern for water waves is shown in **Figure 3.2**. Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. These angles depend on wavelength and the distance between the slits, as we shall see below.

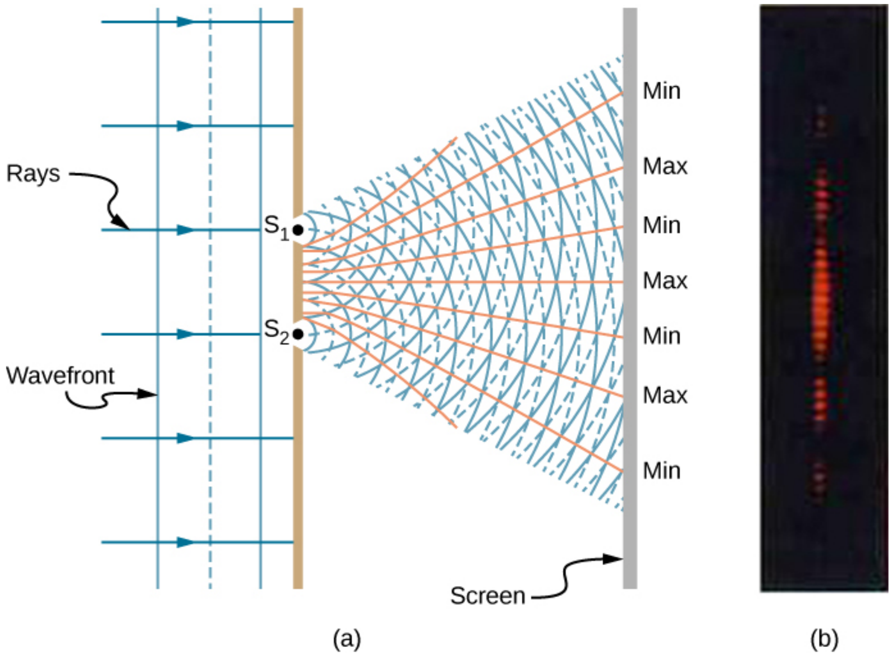

Figure 3.5 Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. These waves overlap and interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) When light that has passed through double slits falls on a screen, we see a pattern such as this.

To understand the double-slit interference pattern, consider how two waves travel from the slits to the screen (**Figure 3.6**). Each slit is a different distance from a given point on the screen. Thus, different numbers of wavelengths fit into each path. Waves start out from the slits in phase (crest to crest), but they may end up out of phase (crest to trough) at the screen if the paths differ in length by half a wavelength, interfering destructively. If the paths differ by a whole wavelength, then the waves arrive in phase (crest to crest) at the screen, interfering constructively. More generally, if the path length difference Δl between the two waves is any half-integral number of wavelengths [$(1/2)\lambda$, $(3/2)\lambda$, $(5/2)\lambda$, etc.], then destructive interference occurs. Similarly, if the path length difference is any integral number of wavelengths (λ, 2λ, 3λ, etc.), then constructive interference occurs. These conditions can be expressed as equations:

$$\Delta l = m\lambda, \quad \text{for } m = 0, \pm 1, \pm 2, \pm 3 \ \ldots \ \text{(constructive interference)} \tag{3.1}$$

$$\Delta l = \left(m + \frac{1}{2}\right)\lambda, \quad \text{for } m = 0, \pm 1, \pm 2, \pm 3 \ \ldots \ \text{(destructive interference)} \tag{3.2}$$

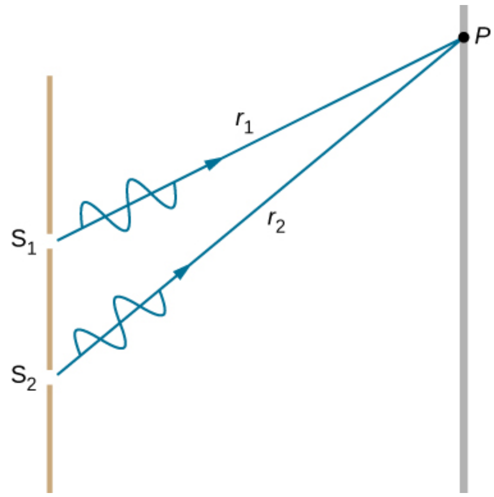

Figure 3.6 Waves follow different paths from the slits to a common point *P* on a screen. Destructive interference occurs where one path is a half wavelength longer than the other—the waves start in phase but arrive out of phase. Constructive interference occurs where one path is a whole wavelength longer than the other—the waves start out and arrive in phase.

3.2 | Mathematics of Interference

Learning Objectives

By the end of this section, you will be able to:

- Determine the angles for bright and dark fringes for double slit interference
- Calculate the positions of bright fringes on a screen

Figure 3.7(a) shows how to determine the path length difference Δl for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle θ between the path and a line from the slits to the screen [part (b)] is nearly the same for each path. In other words, r_1 and r_2 are essentially parallel. The lengths of r_1 and r_2 differ by Δl, as indicated by the two dashed lines in the figure. Simple trigonometry shows

$$\Delta l = d \sin \theta \tag{3.3}$$

where *d* is the distance between the slits. Combining this result with **Equation 3.1**, we obtain constructive interference for a double slit when the path length difference is an integral multiple of the wavelength, or

$$d \sin \theta = m\lambda, \text{ for } m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ (constructive interference).} \tag{3.4}$$

Similarly, to obtain destructive interference for a double slit, the path length difference must be a half-integral multiple of the wavelength, or

$$d \sin \theta = (m + \frac{1}{2})\lambda, \text{ for } m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ (destructive interference)} \tag{3.5}$$

where λ is the wavelength of the light, *d* is the distance between slits, and θ is the angle from the original direction of the beam as discussed above. We call *m* the **order** of the interference. For example, $m = 4$ is fourth-order interference.